DEPARTMENT OF ELECTRICAL ENGINEERING

BASIC ELECTRICAL ENGINEERING (4 credit)
Course Code: 23UEEL1101

Unit II A. C. Circuits (08 HOURS)

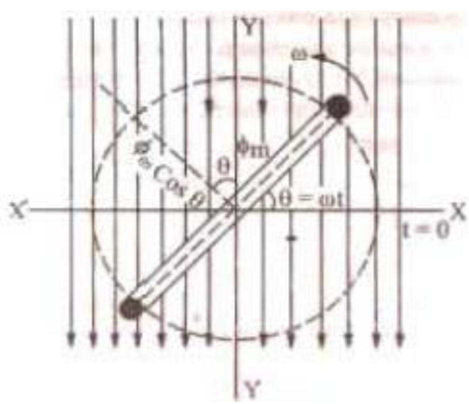
Single phase EMF generation:

Alternating voltage may be generated

1) By rotating a coil in a magnetic field
2) By rotating a magnetic field within a stationary coil

The value of voltage generated depends upon
1) No. of turns in the coil 2) field strength 3) speed

Equation of alternating voltage and current



N= No. of turns in a coil

Φ_m= Maximum flux when coil coincides with X-axis

ω= angular speed (rad/sec) = 2πf

At θ=ωt, Φ= flux component ⊥ to the plane =Φ_m cos ωt

According to the Faraday's law of electromagnetic induction,

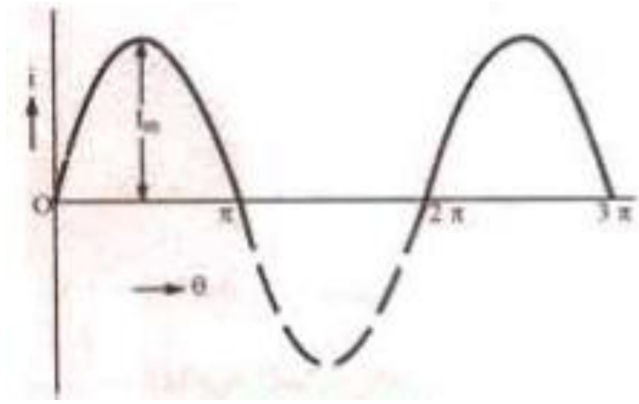
$$e=-N\frac{d\varphi}{dt}=-N\frac{d}{dt}\varphi_m Cos\omega t=\omega N\varphi_m Sin\omega t ....(1)$$

Now, e is maximum value of E_m , when Sinθ = Sin 90° = 1.

i.e $E_m = \omega N\Phi_m$ ....(2)

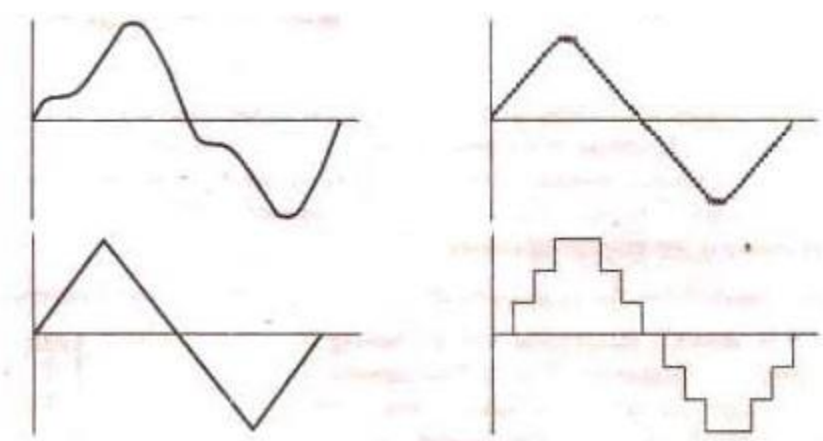
From Eq^n (1) & (2), $e = E_m \sin \omega t$ volt

Now, current (i) at any time in the coil is proportional to the induced emf (e) in the coil. Hence, $i = I_m \sin \omega t$ amp



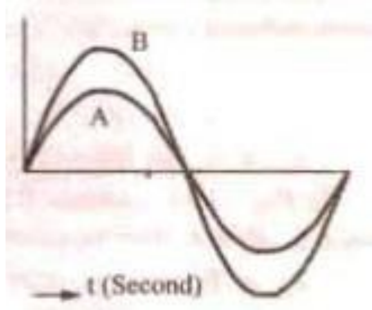
A.C terms:

- Cycle:- A complete set of positive and negative values of an alternating quantity is known as cycle.



- Time period: The time taken by an alternating quantity to complete one cycle is called time T.
- Frequency: It is the number of cycles that occur in one second. $f = 1/T$
  $f = PN/120$ where, P= No. of poles, N= Speed in rpm
- Waveform: A curve which shows the variation of voltage and current w.r.t time or rotation.

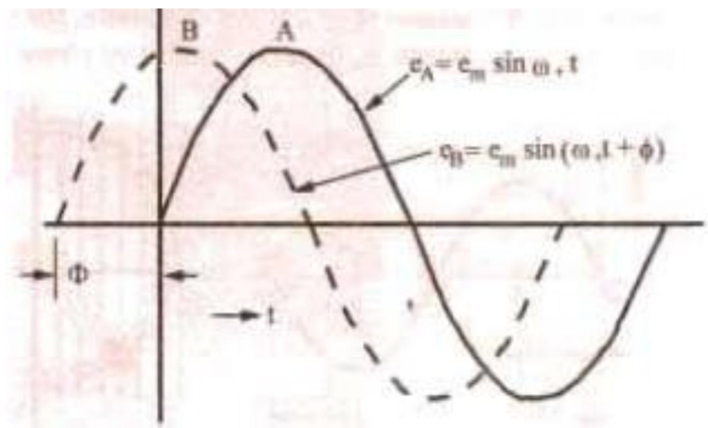
- Phase & Phase difference:



<u>In phase</u>: $e_A = E_{mA} Sin\omega t$

$e_B = E_{mB} Sin\omega t$

<u>Out of phase: </u>i) B leads A



Phase difference Φ. $e_A = E_m Sin\omega t$

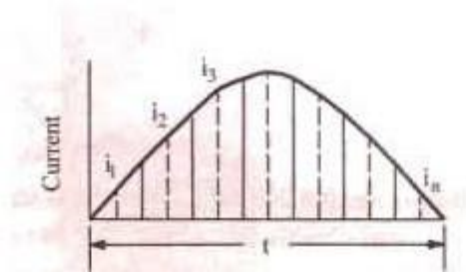
$e_B = E_{mB} Sin(\omega t + \alpha)$

ii) A leads B or B lags A

$e_A = E_m Sin\omega t$

$e_B = E_m Sin(\omega t - \alpha)$

Root mean Square (RMS) or effective or virtual value of A.C:-



$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots\ldots + i_n^2}{n}} = \text{Square root of the mean of square of the instantaneous currents}$$

- It is the square root of the average values of square of the alternating quantity over a time period.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\omega t)d(\omega t)}$$

Average Value (or mean value):

- It is the arithmetic sum of all the instantaneous values divided by the number of values used to obtain the sum

$$I_{av} = \frac{i_1 + i_2 + \ldots\ldots + i_n}{n}$$

$$I_{av} = \frac{1}{T}\int_0^T i(\omega t)d(\omega t)$$

Form factor (K_f):- is the ratio of rms value to average value of an alternating quantity. ($K_f = I_{rms}/I_{av}$)

Peak factor (K_a) or crest factor:- is the ratio of peak (or maximum) value to the rms value of alternating quantity ($Ka = I_{max}/ I_{rms}$)

Example: An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value a) 0.0025 sec b) 0.0125 sec after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

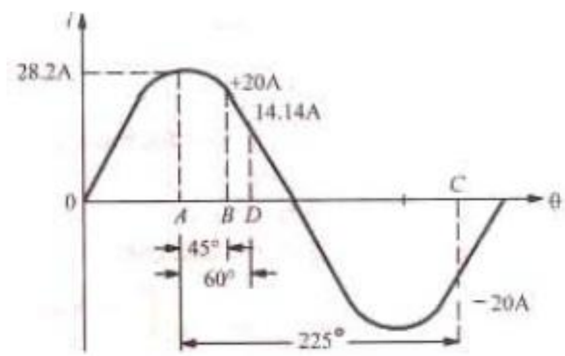
Ans: $I_m = 20\sqrt{2} = 28.2\,A$
$\omega = 2\pi \times 50 = 100\pi \text{ rad/s}$

The equation of the sinusoidal current wave with reference to point O as zero time point is

i= 28.2 sin 100πt Ampere

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In this case, equation becomes

i= 28.2 cos 100πt



i) When t= 0.0025 second
i = 28.2 cos 100π X 0.0025 ....angle in radian
= 28.2 cos 100 X 180 X 0.0025..................angle in degrees
= 28.2 cos 45° = 20 A................................ point B

ii) When t = 0.0125 sec
I =28.2 cos 100 X 180 X 0.0125
= 28.2 cos 225° = 28.2 X (-1/√2)
= -20 A...point C

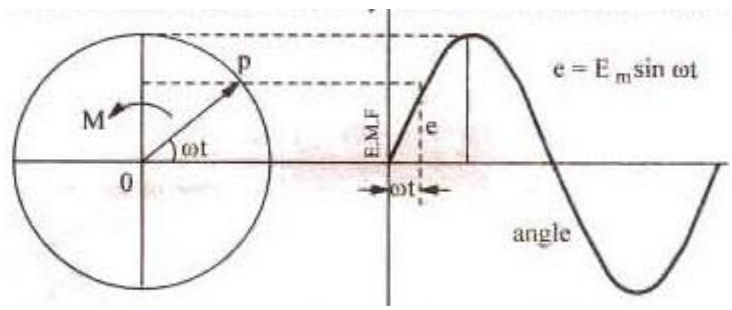
iii) Here $i = 14.14$ A

$14.14 = 28.2 \cos 100 \times 180 t$

$\cos 100 \times 180 t = ½$

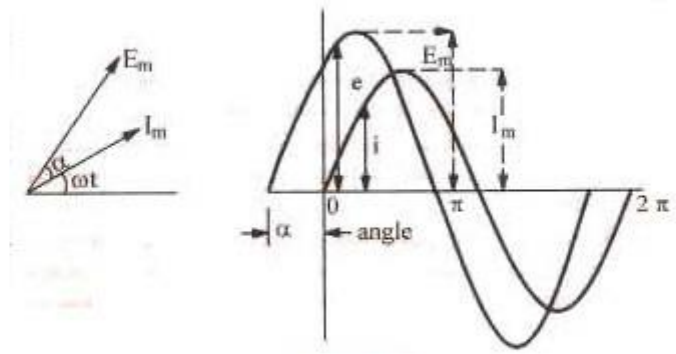
Or, $100 \times 180 t = \cos^{-1}(1/2) = 60°$, $t = 1/300$ sec.................. point D

Phasor & Phasor diagram:



Phasor: Alternating quantities are vector (i.e having both magnitude and direction). Their instantaneous values are continuously changing so that they are represented by a rotating vector (or phasor). A phasor is a vector rotating at a constant angular velocity

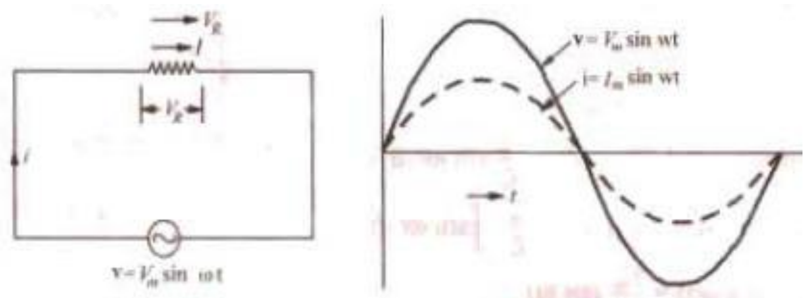
Phasor diagram: is one in which different alternating quantities of the same frequency are represented by phasors with their correct phase relationship



Points to remember:

1. The angle between two phasors is the phase difference
2. Reference phasor is drawn horizontally
3. Phasors are drawn to represent rms values
4. Phasors are assumed to rotate in anticlockwise direction
5. Phasor diagram represents a “still position” of the phasors in one particular point

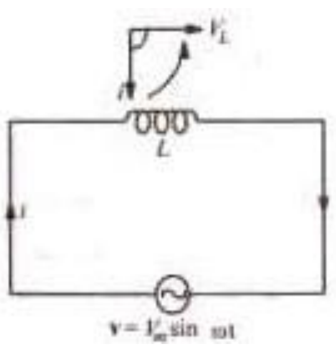
A.C through pure ohmic resistance only

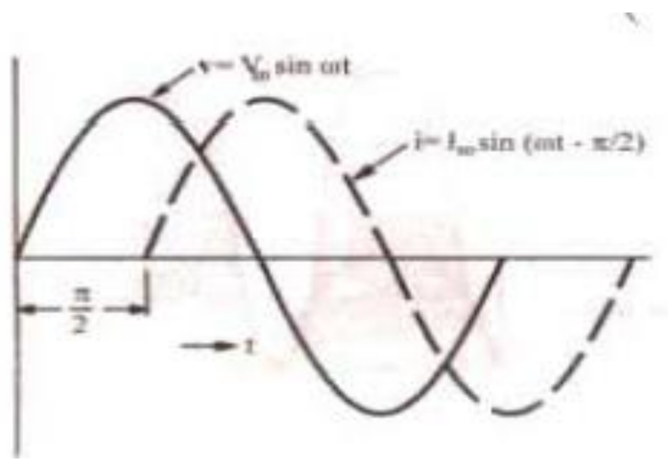


$$v = iR \text{ or } i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t \; (\text{in phase})$$

A.C through pure inductance only

v= L





$$\frac{di}{dt} = V_m \sin\omega t$$

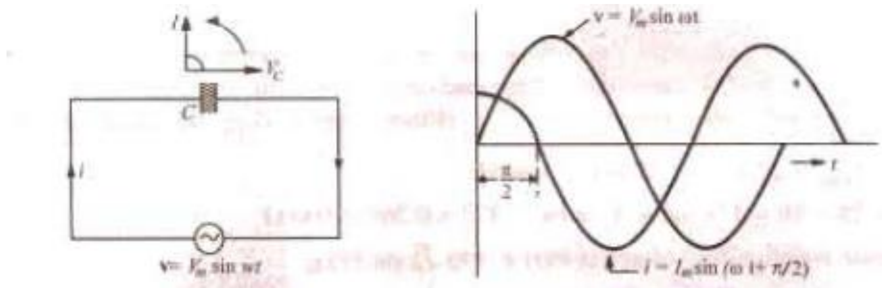
$$i = \frac{V_m}{L}\int \sin\omega t$$

$$i = -\frac{V_m}{\omega L}\cos\omega t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$ (current lags by 90°)

$\omega L = 2\pi f L = X_L$ = inductive reactance(in Ω)

A.C through pure Capacitance only



$$i = C\frac{dv}{dt} = C\frac{d}{dt}\left(V_m \sin\omega t\right)$$

$$= \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{(current leads by } 90°\text{)}$$

$$\frac{1}{\omega C} = X_C = \frac{1}{2\pi f C} = \text{capacitive reactance (in } \Lambda\text{)}$$

'j' operator: j is a operator which rotates a vector by 90° in anticlockwise direction

$$j^2 = -1 ; j = \sqrt{-1}$$

Note: 'i' is used for current hence 'j' is used to avoid confusion

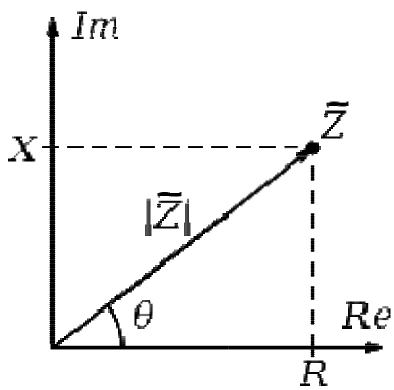
Mathematical representation of vectors:

1. Rectangular or Cartesian form :- $\vec{V} = a \pm jb$
2. Polar form : $\vec{V} = V\angle \pm\theta$
3. Trignometrical form : $\vec{V} = V\left(\cos\theta \pm j\sin\theta\right)$
4. Exponential form : $\vec{V} = V e^{\pm j\theta}$

Note: rectangular form is best suited for addition and subtraction & polar form is best suited for multiplication and division

IMPEDANCE:

In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuit. Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude. When a circuit is driven with direct current (DC), there is no distinction between impedance and resistance; the latter can be thought of as impedance with zero phase angle.



Where X=Total reactance of the network (Both inductive and capacitive)

R=Resistance of the network in ohm.

θ= Phasor angle in degree/Radian.

Note:

I. If θ =0 degree then the load is purely **Resistive**.

II. If θ=-90 degree then the load is purely **inductive**.(lagging)

III. If θ=90 degree then the load is purely **capacitive**.(leading)

Z=R+jX

Where Z=impedance of the electrical network in ohm.

R=Resistance of the network in ohm.

X=Reactance of the electrical network in ohm.

Admittance:

In electrical engineering, admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the inverse of impedance. The SI unit of admittance is the siemens (symbol S).

Admittance is defined as:

$$Y = 1/Z$$

Where

Y is the admittance, measured in siemens

Z is the impedance, measured in ohms

The synonymous unit mho, and the symbol ℧ (an upside-down uppercase omega Ω), are also in common use.

Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (known as reactance). Likewise, admittance is not only a measure of the ease with which a steady current can flow, but also the dynamic effects of the material's susceptance to polarization:

$$Y = G + j B$$

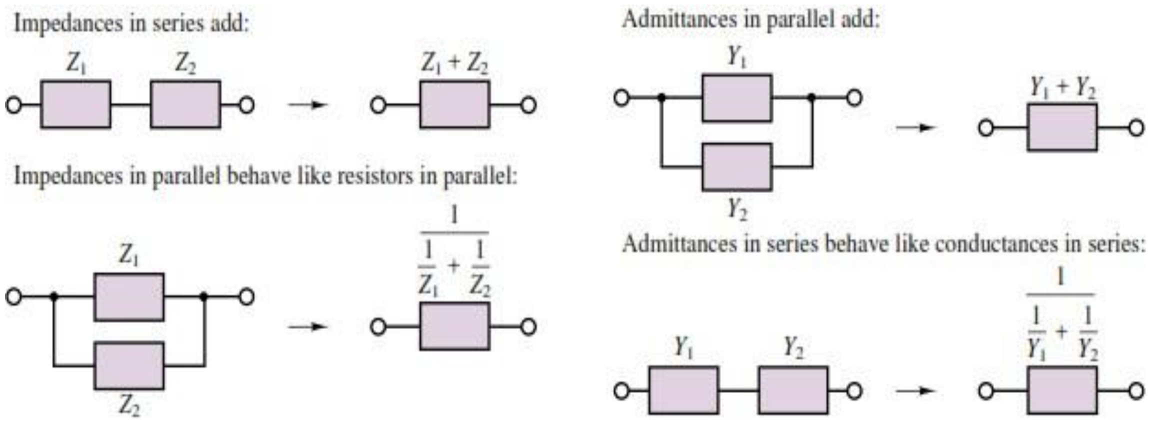
Where

Y is the admittance, measured in siemens.

G is the conductance, measured in siemens.

B is the susceptance, measured in siemens.

AC Equivalent Circuits:



1. Impedances in series add together to give the equivalent impedance while the admittance in parallel add together to give the equivalent admittance.
2. Impedances in parallel gives equivalent impedance by reciprocating the reciprocal sum of the impedances and to obtain the equivalent admittance in series same procedure has to be followed.

Instantaneous and Average Power

The most general expressions for the voltage and current delivered to an arbitrary load are as follows:

$v(t) = V \cos(\omega t - \theta_V)$

$i(t) = I \cos(\omega t - \theta_I)$

Since the instantaneous power dissipated by a circuit element is given by the product of the instantaneous voltage and current, it is possible to obtain a general expression for the power dissipated by an AC circuit element:

$p(t) = v(t)i(t)= V I \cos(\omega t) \cos(\omega t - \theta)$

It can be further simplified with the aid of trigonometric identities to yield

$p(t) = V I/2\cos(\theta) +V I/2 \cos(2\omega t - \theta)$

where θ is the difference in phase between voltage and current

The average power corresponding to the voltage and current signal can be obtained by integrating the instantaneous power over one cycle of the sinusoidal signal. Let $T = 2\pi/\omega$ represent one cycle of the sinusoidal signals. Then the average power, Pav, is given by the integral of the instantaneous power,

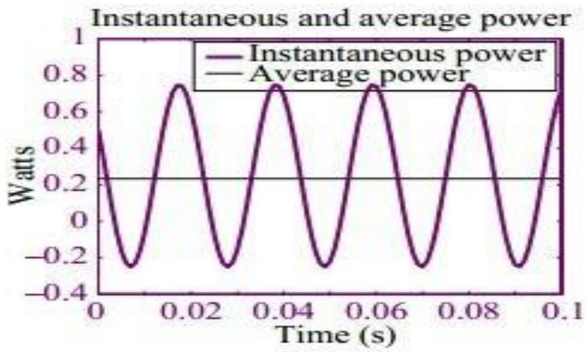
p(t), over one cycle:

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{VI}{2}\cos(\theta)\,dt + \frac{1}{T}\int_0^T \frac{VI}{2}\cos(2\omega t - \theta)\,dt$$

$$P_{av} = \frac{VI}{2}\cos(\theta) \qquad \text{Average power}$$

since the second integral is equal to zero and cos(θ) is a constant.



In phasor notation, the current and voltage are given by

$$\mathbf{V}(j\omega) = Ve^{j\theta}$$
$$\mathbf{I}(j\omega) = Ie^{-j\theta}$$

impedance of the circuit elementdefined by the phasor voltage and currentto be

$$Z = \frac{V}{I}e^{-j(\theta)} = |Z|e^{j\theta_Z}$$

The expression for the average power using phasor notation

$$P_{\text{av}} = \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{1}{2}I^2|Z|\cos\theta$$

Power Factor

The phase angle of the load impedance plays a very important role in the absorption of power by load impedance. The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term cos(θ) is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case,0<pf<1.If the load has an inductive reactance, then θ is positive and the current lags (or follows) the voltage. Thus, when θ and Q are positive, the corresponding power factor is termed lagging. Conversely, a capacitive load will have a negative Q, and hence a negative θ. This corresponds to a leading power factor, meaning that the load current leads the load voltage.A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy .Two equivalent expressions for the power factor are given in the following:

$$\text{pf} = \cos(\theta) = \frac{P_{\text{av}}}{\tilde{V}\tilde{I}} \qquad \text{Power factor}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current.

Complex Power

The expression for the instantaneous power may be further expanded to provide further insight into AC power. Using trigonometric identities, we obtain the following expressions:

$$\begin{aligned} p(t) &= \frac{\tilde{V}^2}{|Z|}[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|\cos\theta(1 + \cos(2\omega t)) + \tilde{I}^2|Z|\sin\theta\sin(2\omega t) \end{aligned}$$

Recalling the geometric interpretation of the impedance Z

$|Z| \cos \theta = R$ and $|Z|\sin \theta = X$

are the resistive and reactive components of the load impedance, respectively. On the basis of this fact, it becomes possible to write the instantaneous power as:

$$p(t) = \tilde{I}^2 R(1 + \cos(2\omega t)) + \tilde{I}^2 X \sin(2\omega t)$$
$$= \tilde{I}^2 R + \tilde{I}^2 R \cos(2\omega t) + \tilde{I}^2 X \sin(2\omega t)$$

Since Pav corresponds to the power absorbed by the load resistance, it is also called the real power, measured in units of watts (W). On the other hand, Q takes the name of reactive power, since it is associated with the load reactance. The units of Q are volt-amperes reactive, or VAR. Note that Q represents an exchange of energy between the source and the reactive part of the load; thus, no net power is gained or lost in the process, since the average reactive power is zero. In general, it is desirable to minimize the reactive power in a load.

The computation of AC power is greatly simplified by defining a fictitious but very useful quantity called the complex power, S:

$$S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^*$$

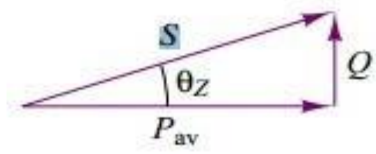
where the asterisk denotes the complex conjugate You may easily verify that this definition leads to the convenient expression

$$S = \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta = \tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or

$$S = P_{av} + jQ$$

The complex power S may be interpreted graphically as a vector in the complex S plane



$$|S| = \sqrt{P_{av}^2 + Q^2} = \tilde{V} \cdot \tilde{I}$$

$$P_{av} = \tilde{V}\tilde{I} \cos \theta$$

$$Q = \tilde{V}\tilde{I} \sin \theta$$

The magnitude of S, |S|, is measured in units of volt-amperes (VA) and is called apparent power, because this is the quantity one would compute by measuring the rms load voltage and currents without regard for the phase angle of the load.The complex power may also be expressed by the product of the square of the rms current through the load and the complex load impedance:

$$S = \tilde{I}^2 Z$$

or

$$\tilde{I}^2 R + j\tilde{I}^2 X = \tilde{I}^2 Z$$

or, equivalently, by the ratio of the square of the rms voltage across the load to the complex conjugate of the load impedance:

$$S = \frac{\tilde{V}^2}{Z^*}$$

Active, Reactive and Apparent Power

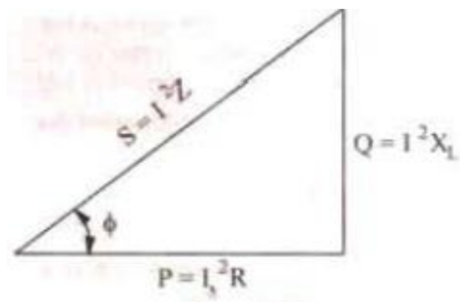


Fig. Power Triangle

$$S^2 = P^2 + Q^2$$

$$S = P + jQ$$

- **Apparent power, S**: is the product of rms values of the applied voltage and circuit current. It is also known as wattless (idle) component
  $S=VI=IZx\ I=I^2Z$ volt-amp
- **Active power or true power, P**: is the power which actually dissipated in the circuit resistance. It is also known as wattful component of power.
  $P=I^2R=I^2Z\cos\Phi=VI\cos\Phi$ watt
- **Reactive power, Q**:- is the power developed in the reactance of the circuit.
  $Q=I^2X=I^2Z\sin\Phi=VI\sin\Phi$ VAR

Example: In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 Ma while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit.

Ans. i)

$$Z=\sqrt{R^2+(2\pi\times 50L)^2}=\sqrt{R^2+98696L^2}$$

$$V=IZ \text{ or } 10=700\times 10^{-3}\sqrt{(R^2+98696L^2)}$$

$$\sqrt{(R^2+98696L^2)}=10/700\times 10^{-3}=100/7$$

$$\text{or } R^2+98696L^2=10000/49..................(i)$$

ii) In the second case

$$Z = \sqrt{R^2 + (2\pi \times 75L)^2} = \sqrt{R^2 + (222066L^2)}$$

$$10 = 500 \times 10^{-3} \sqrt{(R^2 + 222066L^2)}$$

$$\sqrt{(R^2 + 222066L^2)} = 20$$

$$R^2 + 222066L^2 = 400 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

subtracting eq(i) from eq(ii), we get

$$222066L^2 - 98696L^2 = 400 - (10000/49)$$

$$123370L^2 = 196$$

$$L = 0.0398H = 40mH$$

Substituting this value of L in eq(ii),we get

$$R^2 + 222066(0.398)^2 = 400$$

$$R = 6.9\,\Omega$$

Introduction to resonance in series & parallel circuit

Resonance:

Definition: An AC circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So, the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

Series Resonance: In R-L-C series circuit, both X_L and X_C are frequency dependent. If we vary the supply frequency then the values of X_L and X_C varies. At a certain frequency called resonant frequency (f_r), X_L becomes equal to X_C and series resonance occurs.

At series resonance, $X_L = X_C$

$2\pi f_r L = 1/2\pi f_r C$

$f_r = 1/2\pi\sqrt{LC}$

Impedance of RLC series circuit is given by:

$$Z=\sqrt{R^2+(X_L-X_C)^2} \quad (Since, X_L=X_C)$$

$$Z=\sqrt{R^2}$$

$$Z=R$$

$$\cos\varphi=\frac{R}{Z}=\frac{R}{R}=1$$

Properties of series resonance:-

In series resonance,

- The circuit impedance Z is minimum and equal to the circuit resistance R.
- The circuit current I= V/Z = V/R and the current is maximum
- The power dissipated is maximum, $P=V^2/R$
- Resonant frequency is $f_r = 1/2\pi\sqrt{LC}$
- Voltage across inductor is equal and opposite to the voltage across capacitor
- Since power factor is 1, so zero phase difference. Circuit behaves as a purely resistive circuit.

Example: A series RLC circuit having a resistance of 50Λ, an inductance of 500 mH and a capacitance of 400 μF, is energized from a 50 Hz, 230 V, AC supply. Find a) resonant frequency of the circuit b) peak current drawn by the circuit at 50 Hz and c) peak current drawn by the circuit at resonant frequency

Ans.

$$a)f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500\times10^{-3}\times400\times10^{-6}}} = 11.25\ Hz$$

a) $R=50\Omega$

$$X_L = \omega L = 2\pi\times50\times500\times10^{-3} = 157\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times50\times400\times10^{-6}} = 7.9\Omega$$

$$X = X_L - X_C = 157-7.9 = 149.1\Omega$$

$$Z = \sqrt{R^2+X^2} = \sqrt{50^2+149.1^2} = 157.26\Omega$$

Peak supply voltage, $V_m = \sqrt{2}\ V_{rms} = \sqrt{2}\ (230) = 325.26\ V$

Hence peak current at 50Hz $I_m = \frac{V_m}{Z} = \frac{325.26}{157.26} = 2.068$

b) At resonance, $Z_0 = R = 50\Omega$

So, peak current during resonance, $I_{mo} = \frac{V_m}{R} = \frac{325.26}{50} = 6.5025A$

Parallel resonance:

Points to remember:

- Net susceptance is zero, i.e $1/X_C = X_L/Z^2$

X_L x $X_C = Z^2$

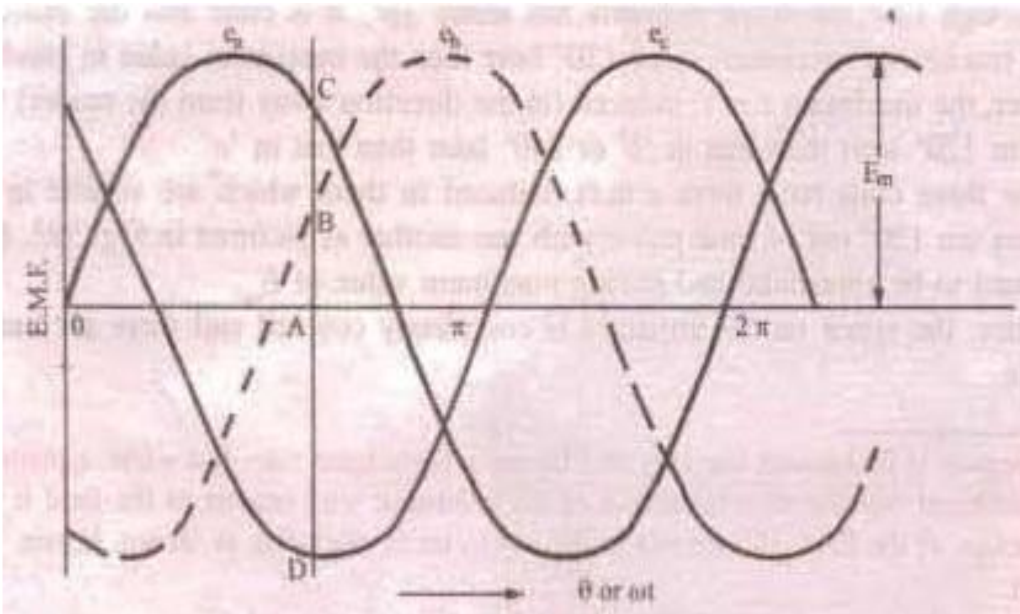
Or $L/C = Z^2$

- The admittance equals conductance
- Reactive or wattless component of line current is zero
- Dynamic impedance = L/CR Λ

- Line current at resonance is minimum and V/L/CR but is in phase with the applied voltage
- Power factor of the circuit is unity

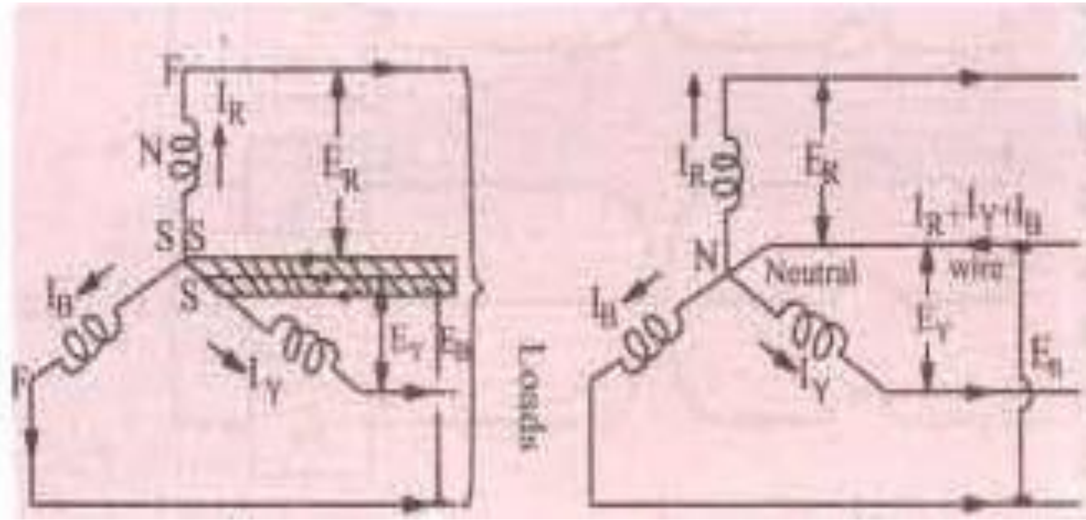
THREE PHASE AC CIRCUIT

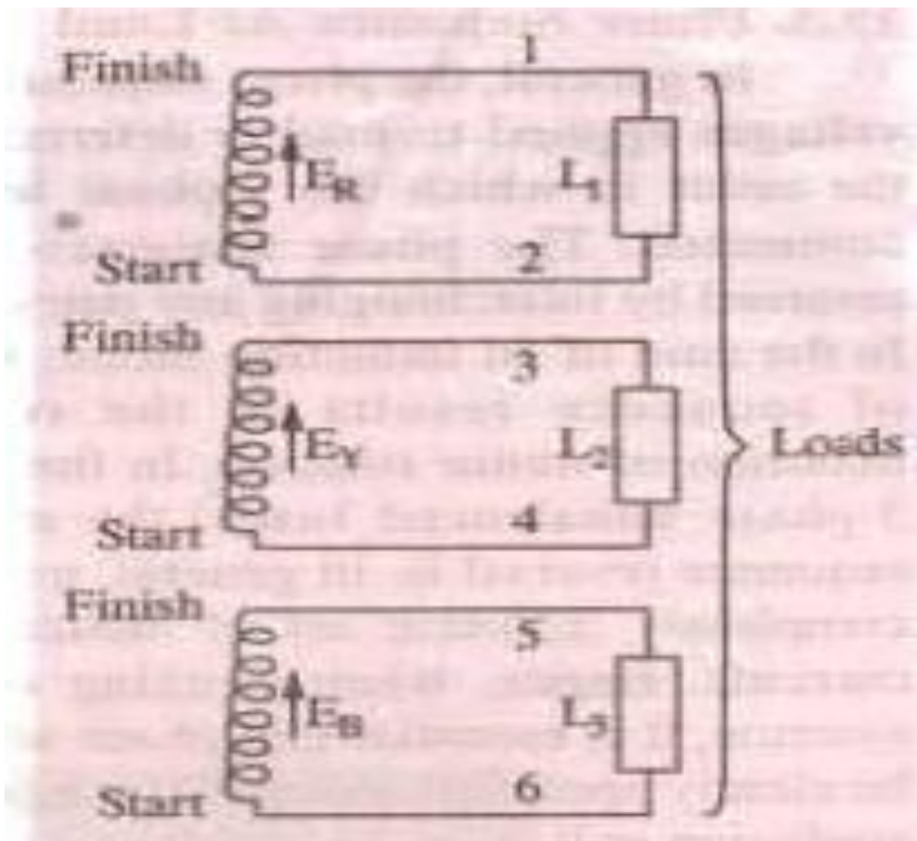
Three phase EMF Generation:-

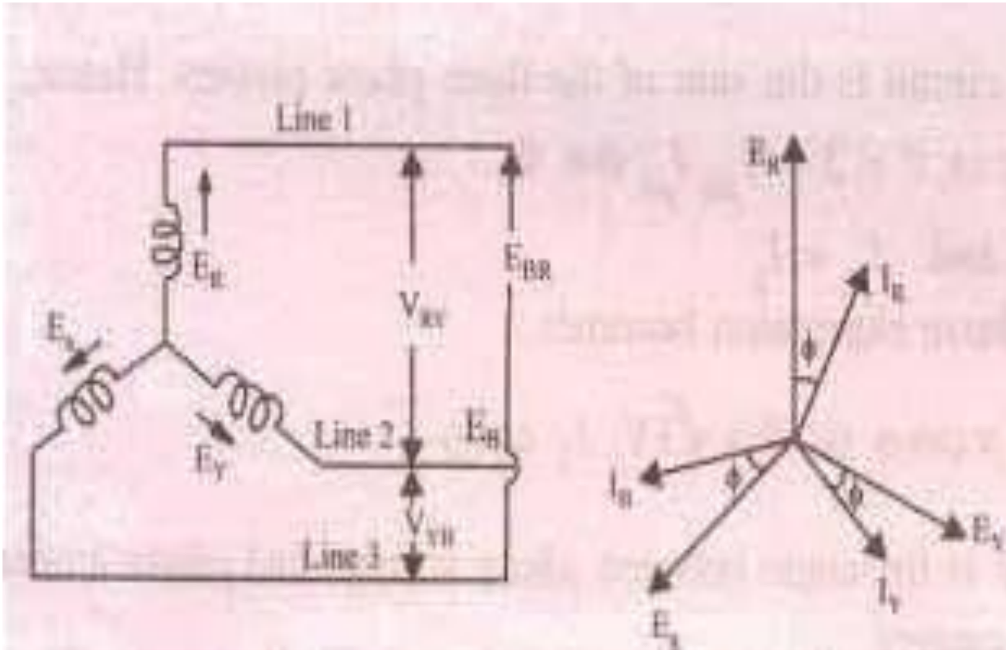


If the 3-coil windings W_1, W_2 and W_3 arranged at 120° apart from each other on the same axis are rotated, then the emf induced in each of them will have a phase difference of 120°. In other words if the emf (or current) in one winding (w_1) has a phase of 0°, then the second winding (w_2) has a phase of 120° and the third (w_3) has a phase of 240°.

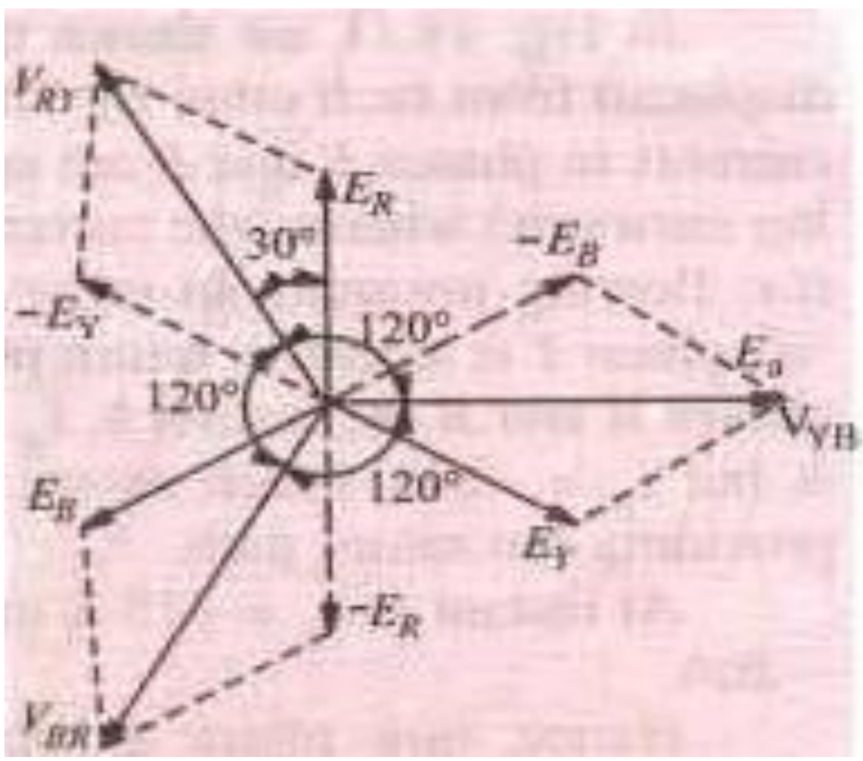
Star (Y) connection:-







Phasor diagram:-



Here, E_R, E_Y, E_B are phase voltages and V_{RY}, V_{YB}, V_{BR} are line voltages

$$V_{RY} = \sqrt{E_R^{\ 2} + E_Y^{\ 2}} + 2E_R E_Y \cos 60°$$

$$= \sqrt{E_R^{\ 2} + E_R^{\ 2}} + 2E_R E_R \cos 60°$$

$$= \sqrt{3} E_R$$

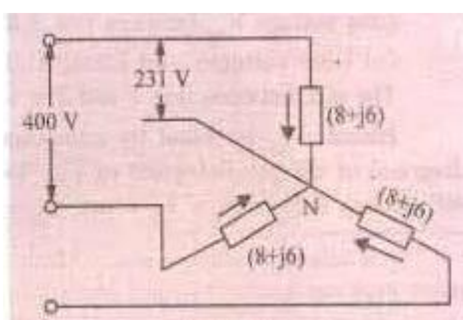
Hence,

- Line voltage =√3 x phase voltage
- Line current = phase current
- Line voltages are also 120° apart
- Line voltage are 30° ahead of respective phase voltages
- The angle between line voltage and line current is (30°+Φ)

Power: Total power = 3 x phase power
=3 x V_{ph} x I_{ph} x cosΦ
=√3 V_L I_L cosΦ
Φ is the angle between phase voltage and current

Example: A balanced star connected load of (8+j6)Λ per phase is connected to a balanced 3-phase 400 V supply. Find the line current, power factor, power and total volt-amperes.

Ans.



$$Z_{ph} = \sqrt{8^2 + 6^2} = 10\Lambda$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 231\,V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1\ A$$

(i) $I_L = I_{ph} = 23.1\ A$

(ii) $p.f = \cos\Phi = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8(lag)$

(iii) Power $P = \sqrt{3}\ V_L I_L \cos\Phi$

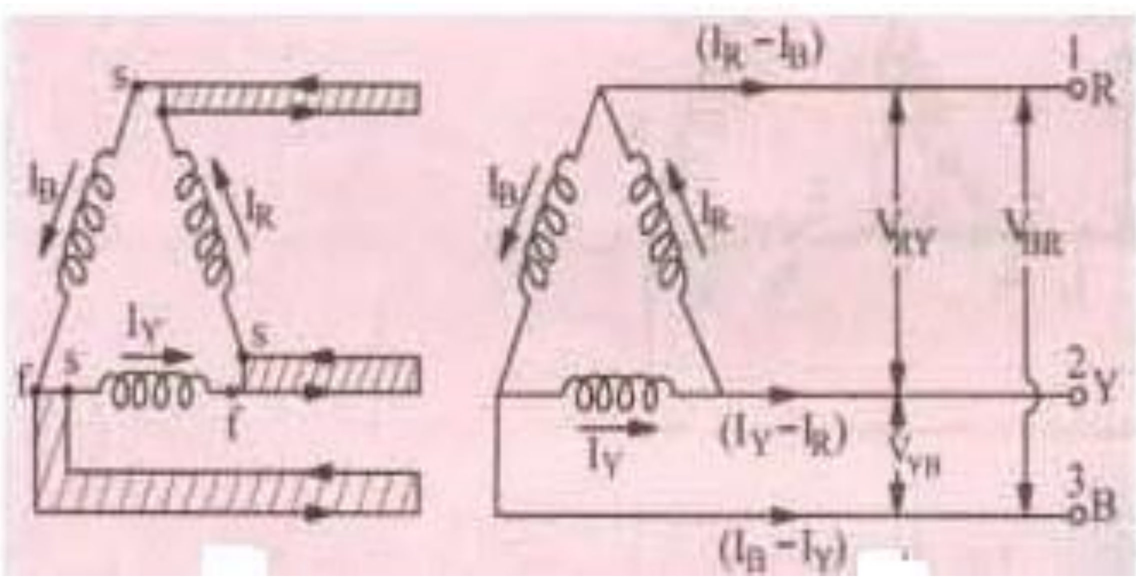
$= \sqrt{3} \times 400 \times 23.1 \times 0.8$

$= 12{,}800\ W$ [Also, $P = 3I_{ph}{}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800\ W$]

(iv) Total volt-amperes,

$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 400 \times 23.1 = 16{,}000\ VA$

<u>Delta-connection</u>:



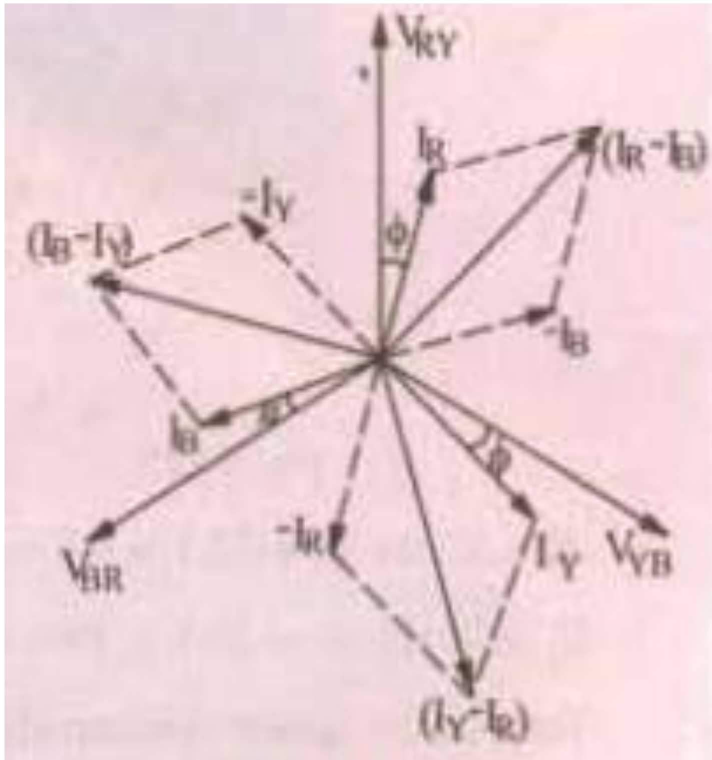


Fig. Phasor Diagram

$I_L = I_R - I_B$

$$I_L = \sqrt{I_R^{\ 2} + I_B^{\ 2} + 2 I_R\, I_B \cos 60^o} = \sqrt{I_R^{\ 2} + I_R^{\ 2} + 2 I_R\, I_R \cos 60^o} = \sqrt{3} I_R$$

Hence,

- Line current =√3 phase current
- Line voltage = phase voltage
- Line currents are also 120° apart
- Line currents are 30° behind the respective phase currents
- Angle between line current and line voltage is 30°+Φ

Power: Total power = 3 x phase power

$= 3 \times V_{ph}\, I_{ph} \cos\Phi$

$= 3 \times V_L \times I_L/\sqrt{3} \times \cos\Phi$

$= \sqrt{3}\, V_L\, I_L \cos\Phi$

Note: For both star and delta system:

Active & True power = $\sqrt{3}\ V_L\ I_L\ \cos\Phi$

Reactive power =$\sqrt{3}\ V_L\ I_L\ \sin\Phi$

Apparent power =$\sqrt{3}\ V_L\ I_L$